Studies of Gyrokinetic Turbulence Models for Edge Plasmas

E. A. Belli*, J. Candy, P. B. Snyder

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edge
Simulation
Laboratory

GENERAL ATOMICS

EGK: A prototype Eulerian gyrokinetic code for ESL based on the (μ,$v_{||}$) velocity space formulation

First as δf to explore numerical dissipation algorithms for (μ,$v_{||}$)

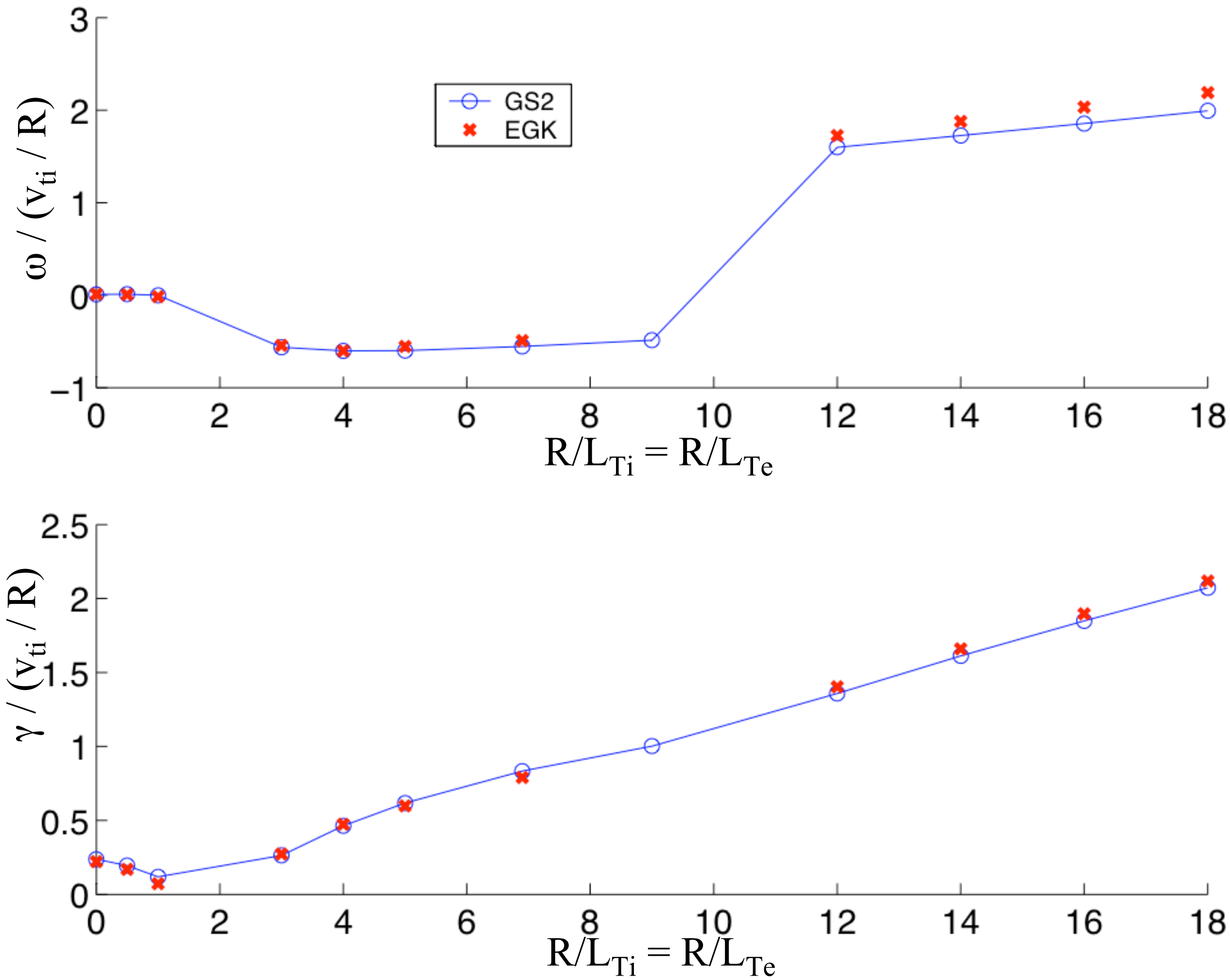
$$\frac{\partial f}{\partial t} + \left(v_{\parallel}\hat{b}\cdot\nabla + i\omega_{dv} + \delta\vec{v}_E\cdot\nabla - \boxed{\mu\hat{b}\cdot\nabla B\frac{\partial}{\partial v_{\parallel}}} \right) f = \left(i\omega_{*T} - v_{\parallel}\hat{b}\cdot\nabla - i\omega_{dv} \right) \frac{ZeF_M}{T_0} J_0\left(\frac{k_{\perp}v_{\perp}}{\Omega}\right)\Phi$$

$$\Phi\sum_s \frac{Z_s^2 e^2 n_{0s}}{T_{0s}}\left(1-\Gamma_{0s}\right) = \sum_s Z_s e\int d^3v J_{0s}\left(\frac{k_{\perp}v_{\perp}}{\Omega}\right) f_s$$

	Existing δf (R,μ,$v_{||}$) nonlinear gyrokinetic codes:		
	F. Jenko[1]	T. Watanabe & H. Sugama[2] (adiabatic electrons)	**EGK** (linear)
Time integration	3rd order RK	4th order RK-Gill	4th order RK
Phase space derivatives	4th & 5th order compact finite differences	4th & 5th order finite differences	3rd order upwind ($\partial f/\partial\theta$) & 2nd order centered differences

1) CPC **125**, 196 (2000), CPC **163**, 67 (2004), Plas. Phys. Cont. Fus. **47**, B195 (2005); 2) Nucl. Fus. **46**, 24 (2006)

Benchmarks with GS2 in the linear, collisionless, electrostatic limit, including gyrokinetic electrons and trapped particle dynamics, have been successful.

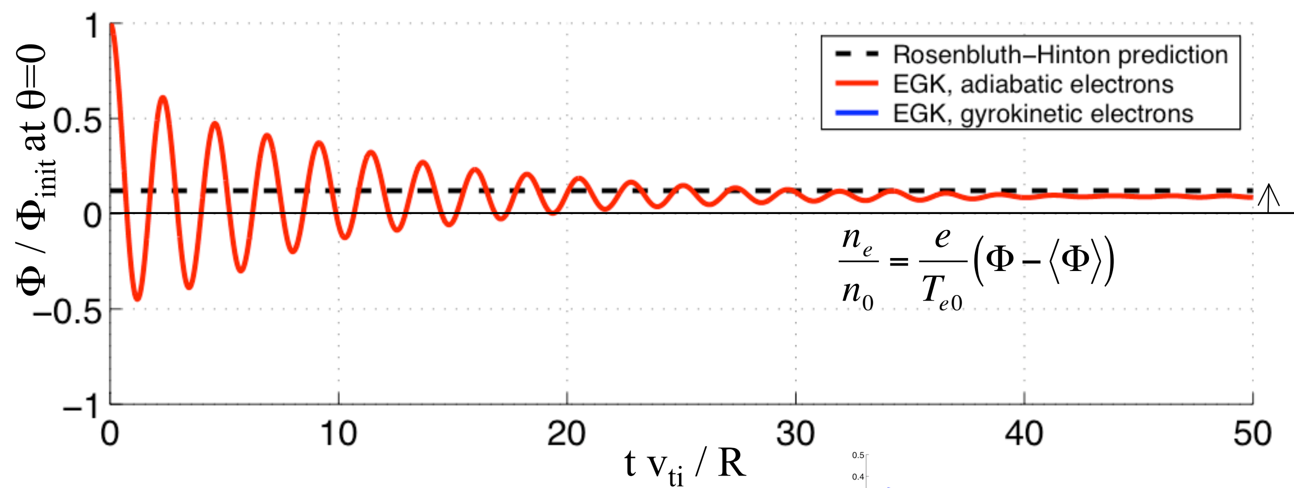


EGK velocity grid:
$\mu/(v_{ts}^2/2B_0) \in [0,25]$
$n\mu$ =21
$v_{||}/v_{ts} \in [-5,5]$
$nv_{||}$=41

GS2: **$n\lambda$=37, nE=16**

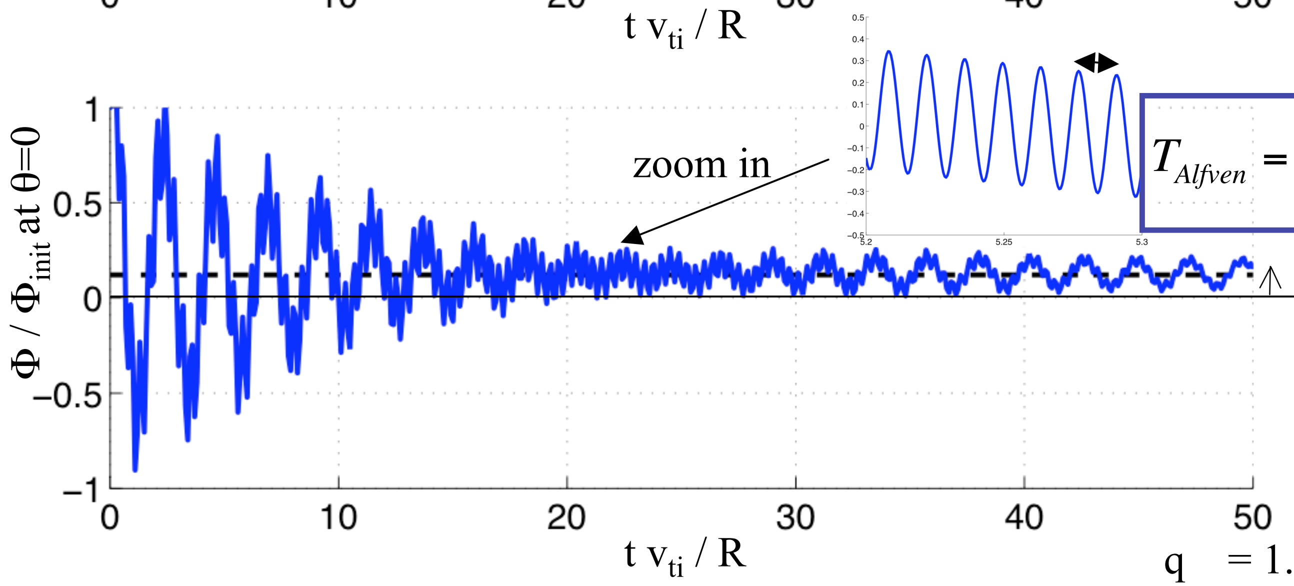
DIII-D Cyclone Base Case Parameters:
s-α geometry
r/R=0.18
q=1.388
s=0.8
α=0
$R/L_{ni}=R/L_{ne}=2.2$
$T_{0i}=T_{0e}$
$k_y\rho_i = 0.4$

Tests of the collisionless damping of the zonal flow potential were also successful.



According to the theory of Rosenbluth & Hinton (Phys.Rev.Lett. **80**, 724 (1998)):

$$\boxed{\lim_{t\to\infty}\frac{\Phi}{\Phi_0} = \frac{1}{1+1.6/h} \qquad h = \sqrt{\varepsilon}/q^2}$$

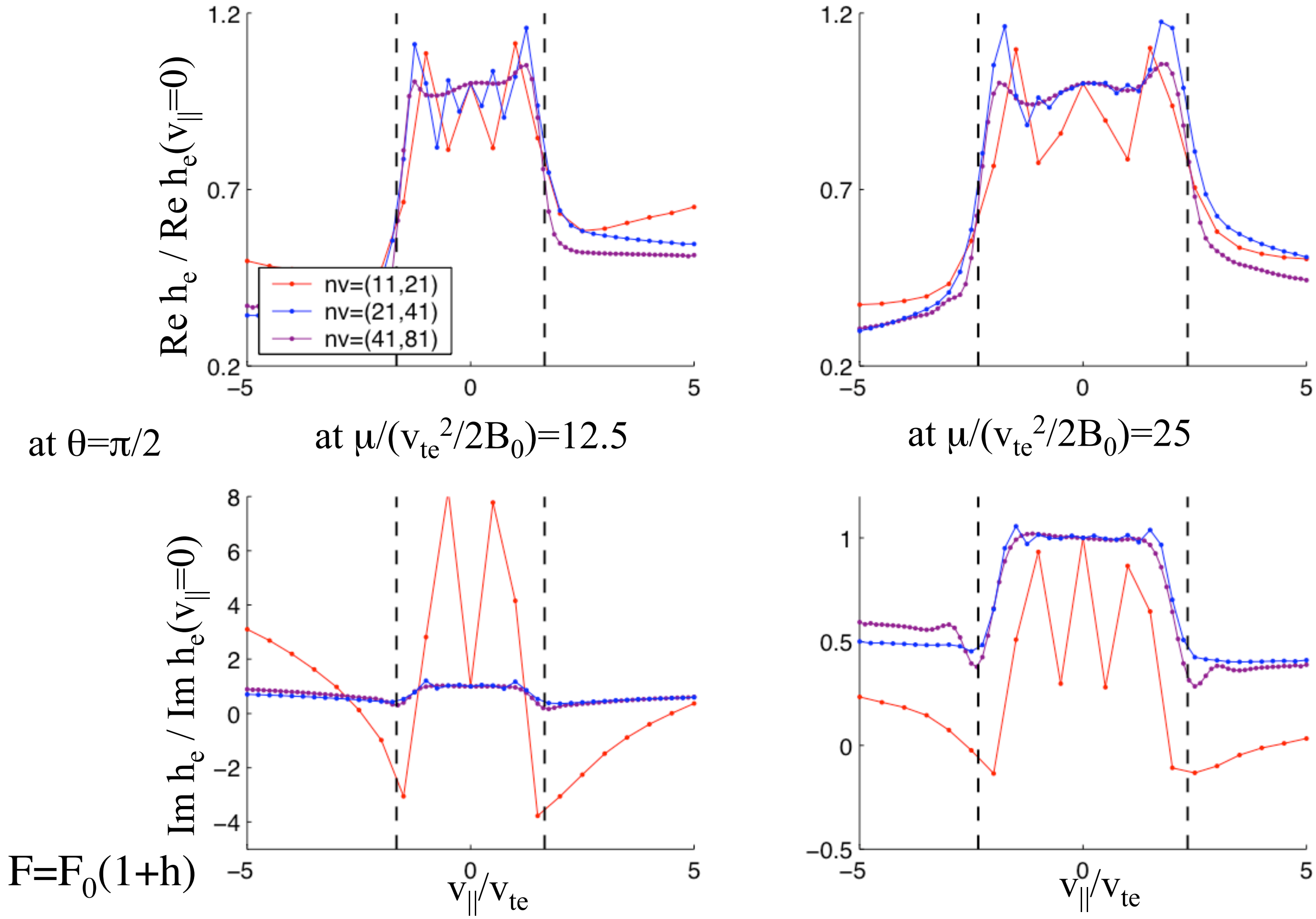


$$\boxed{T_{Alfven} = 2\pi\frac{k_\perp \rho_s}{k_\parallel v_{te}}}$$

q = 1.388, r/R = 0.18, $k_x\rho_i$= 0.1

EGK has been used to study the behavior of f across the trapped/passing particle boundary.

We find that high resolution is needed to reduce numerical oscillations in the trapped region.

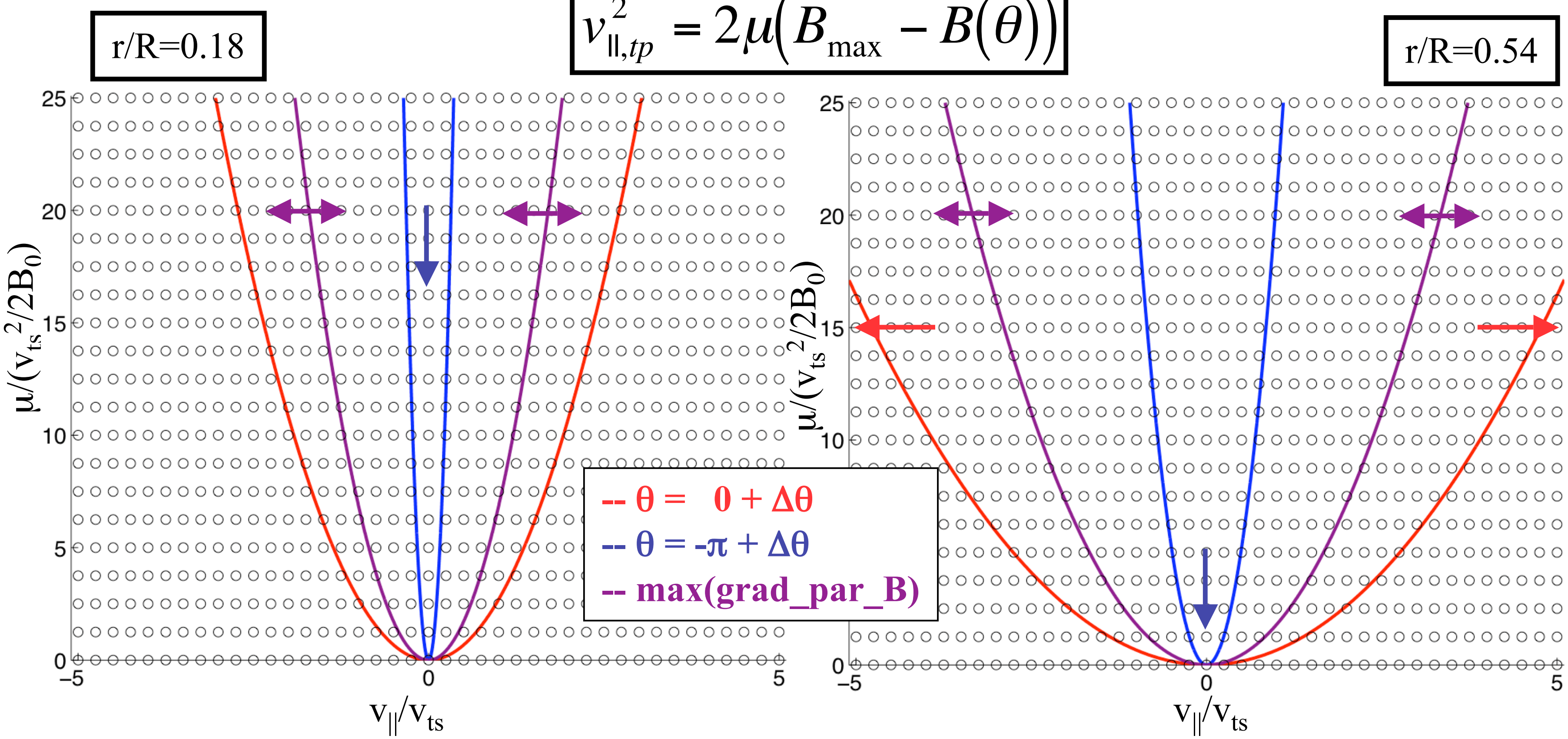


A better approach for the treatment of the parallel velocity derivative across the boundary is needed.

Issues:

1) **Stepping over the boundary when computing the derivative**
2) **One or few points in the trapped region**
3) **None or few points in the passing region**

$$\boxed{v_{\parallel,tp}^2 = 2\mu\left(B_{\max} - B(\theta)\right)}$$



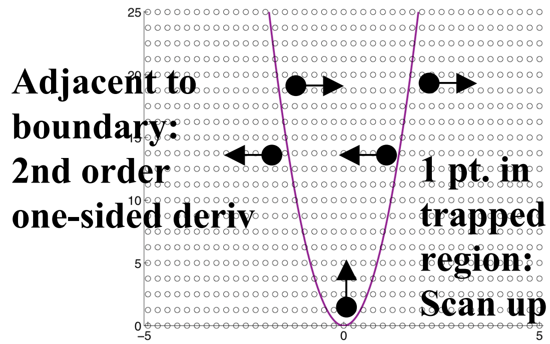
Schemes based on a one-sided derivative for points adjacent to the boundary are numerically unstable.

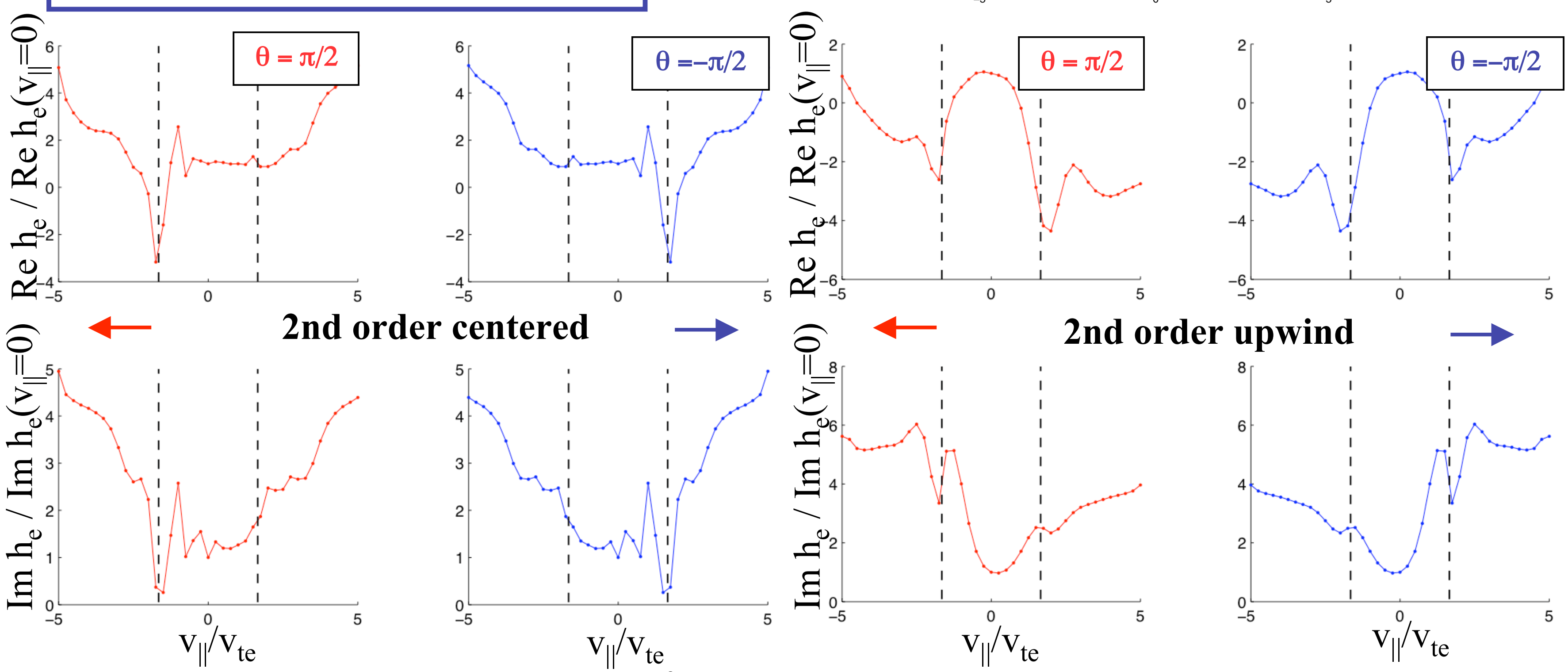
$\theta = \pi/2:$ $-\mu\hat{b} \bullet \nabla B < 0$

$\rightarrow$ positive upwind unstable

$\theta = -\pi/2:$ $-\mu\hat{b} \bullet \nabla B > 0$

$\rightarrow$ negative upwind unstable

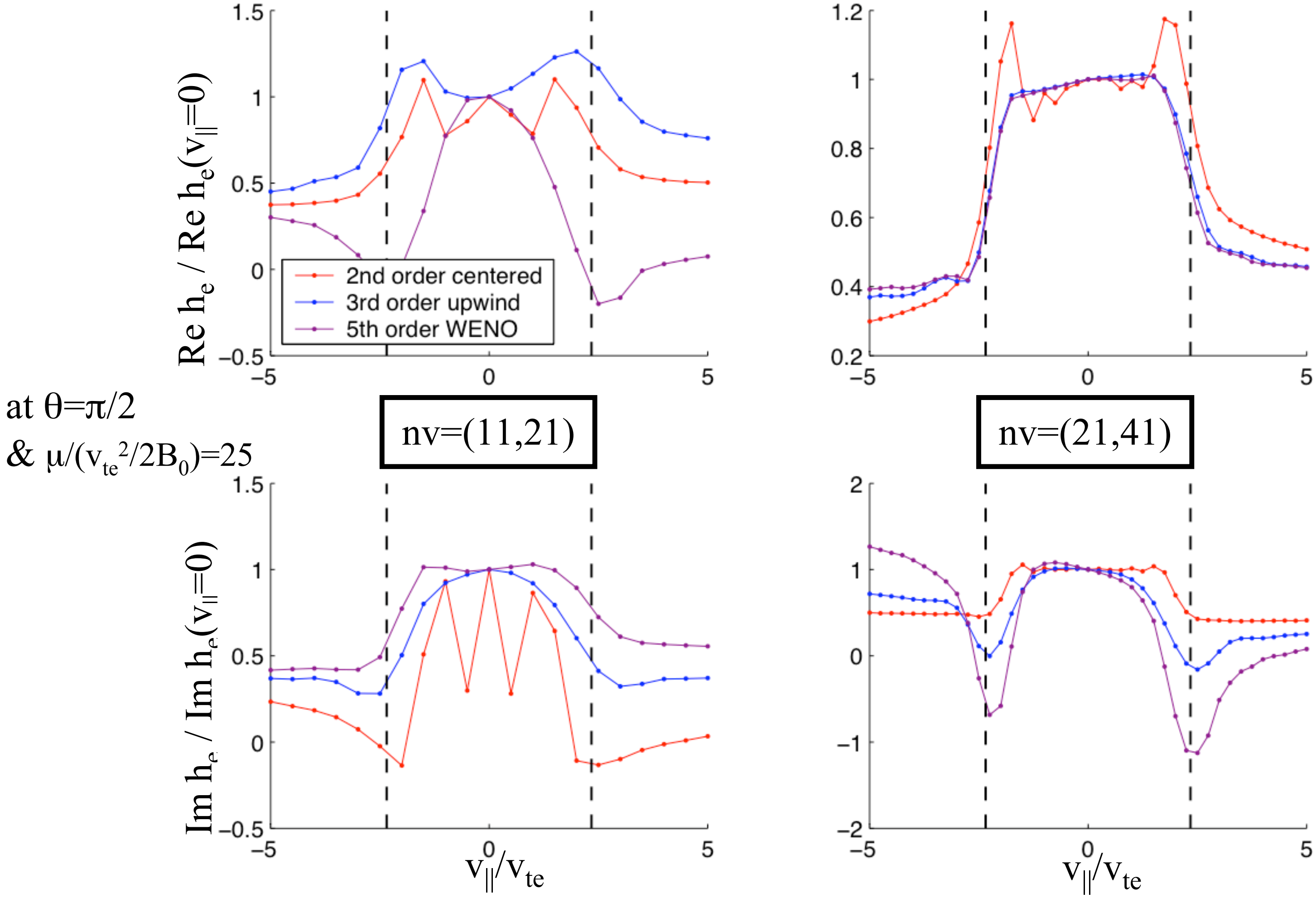




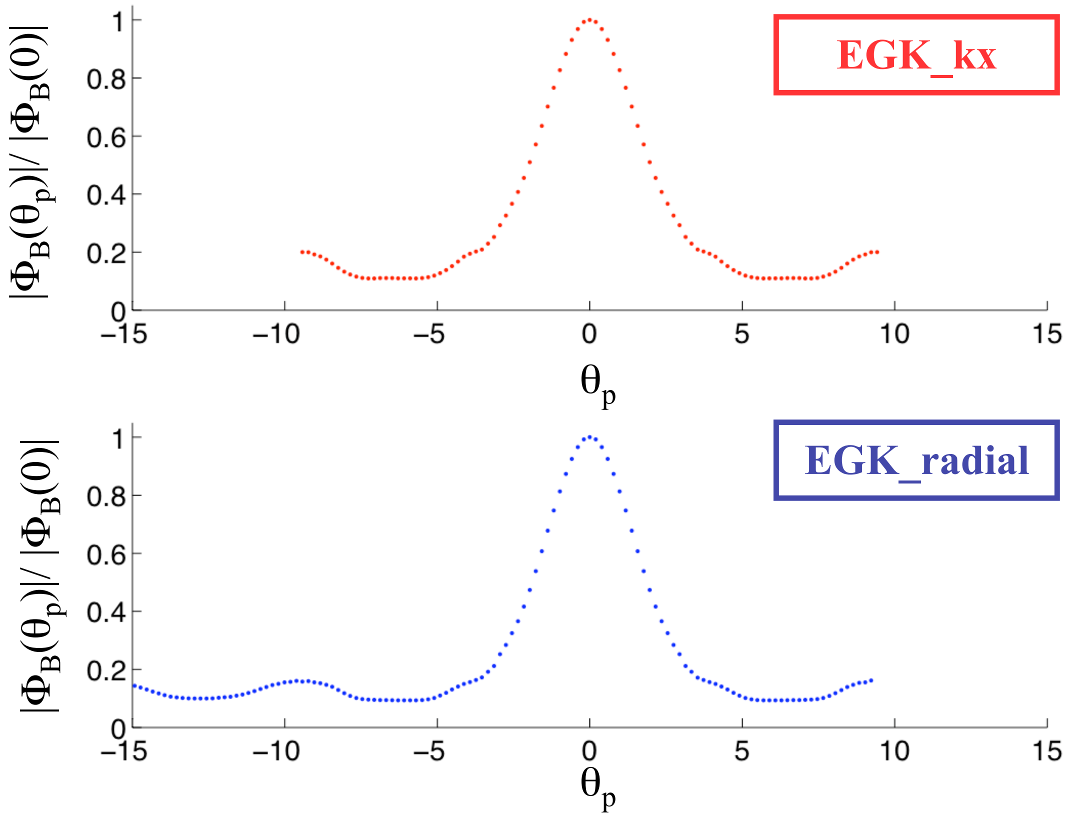
at $\mu/(v_{te}^2/2B_0)$=25 & nv=(21,41)

For now, the best approach is a standard higher-order upwind scheme.

Consider modifications of WENO for sharpening the discontinuities (e.g. subcell resolution, artificial compression, etc.) for full F.



EGK has recently been upgraded to include a radial grid. ITG/TEM and n=0 tests (including gyrokinetic electrons) have been completed and were successful.



- Assumes radially periodic b.c.'s
- Neglects equilibrium radial profile variation

- 2nd order centered scheme for the r derivative of f and Φ (in curvature drift term)
- 3rd order upwind scheme for the $v_{||}$ derivative of f
- 3rd order Heun-RK
- CLAPACK LU decomposition routine for field solver

Summary of EGK: A prototype gyrokinetic code for ESL based on the $(\mu, v_{||})$ velocity space formulation

Motivation:

- **explore numerical issues associated with $(\mu, v_{||})$**
- **study physical effects associated with extensions to full F, such as the parallel nonlinearity**

Present Status:

- $\delta f\ (r, \theta, k_y, \mu, v_{||})$, linear, electrostatic, collisionless
- Includes gyrokinetic electrons and trapped particle dynamics
- Numerical dissipation in θ and $v_{||}$ (3rd order upwind)
- Non-dissipative centered finite difference schemes for the radial derivative and derivatives of Φ
- Explicit 3rd order Heun-Runge-Kutta time stepping integration

- **Velocity space dissipation algorithms have been explored**
- **Successful tests of ITG/TEM linear drift wave physics & collisionless damping of zonal flows completed**

Future Work:

Moving toward full F

$$\frac{\partial (BF)}{\partial t} + \nabla \cdot \left(BF\dot{\vec{R}} \right) + \frac{\partial}{\partial v_{\parallel}} \left(BF\dot{v}_{\parallel} \right) = 0$$

where

$$\dot{\vec{R}} = v_{\parallel}\hat{b} + \vec{v}_d + \vec{v}_E$$

$$\dot{v}_{\parallel} = -\frac{e}{m}\hat{b} \cdot \nabla\Phi - \mu\hat{b} \cdot \nabla B + \frac{v_{\parallel}}{B}\vec{v}_E \cdot \nabla B$$

Where we are headed next: Neoclassical Transport

• Update EGK to include a collision operator. Separate F into a Maxwellian component + a perturbed component δf resulting from magnetic drifts and spatial inhomogeneities. Steady-state simulation for δf.

• Builds on earlier work by Lin et al (Phys. Plasmas **2**, 2975 (1995)) and more recent work by Wang et al (Phys. Plasmas **13**, 082501 (2006)) which explored an extension of standard neoclassical theory to allow for finite orbit widths and including the self-consistent E_r (GTC-Neo).